

Design of random and sparse metalens with matrix pencil method

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Abstract: We propose a matrix pencil method for designing one- or two- dimensional (1D or 2D) metalenses with randomly distributed meta-atoms. In contrast to the standard random synthesis algorithm that only randomizes the position of the meta-atoms, the proposed method designs both the position and phase of each meta-atom rigorously. Several all-dielectric random metalenses, in both 1D and 2D operating at 220 GHz, are presented by using our proposed algorithm. Minimum reduction of focusing efficiency can be achieved with respect to a standard metalens with periodically arranged meta-atoms. In contrast to previously reported random metalenses, our random metalenses achieve much higher efficiency, while staying polarization-independent. This synthesis method will pave a way for future random-metasurface-based device designs, which could have more degrees of freedom to information multiplexing.

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1. Introduction

Metasurfaces are widely designed for wavefront shaping at microwave and optical frequencies [1–4], such as antennas [5–9], radar cross section (RCS) absorbers [10–15], cloaks [16–20], holograms [21,22], flat lenses [23,24], achromatic metalenses [25–27], etc. Wavefront shaping can be obtained by manipulating the phase shifts on the metasurface [28,29]. The building blocks - i.e. meta-atoms - of the metasurface can be metallic or dielectric, and the phase shifts can be obtained by leveraging Pancharatnam-Berry phase [28,29], localized resonances [17,30], waveguide modes [24,31], effective refractive indices [32–34], etc.

When designing a metasurface, the phase profile with respect to the positions required for a specific application will be first calculated, then a set of meta-atoms covering 2π phase shift range will be placed in a periodic grid to construct the target phase profile. The phase error [35] will be produced during the design process: Numerical design of the meta-atoms are usually done using periodic boundary conditions-assuming that the neighboring meta-atoms are identical. However, it is not true for most of the cases. It leads to unexpected phase error, especially where strong near-field coupling between the adjacent meta-atoms exists [35,36].

Recently, a new type of metasurfaces is proposed, where the meta-atoms are placed randomly on a plane [37,38]. In contrast to the conventional design methods which result in highly non-uniform distances between the neighboring meta-atoms, the random metasurface design method predefines a minimum distance between the neighboring ones. We can have uniform distances between the neighboring meta-atoms. These distances can be larger than the minimum distance in a regular metasurface design, and thus reduce the effect of the near-field coupling. In addition, by randomizing the meta-atoms, the metasurface can be polarization-independent. However, it is difficult to rigorously design a random metasurface

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due to the large parameter pace. Here, we present a matrix pencil method that can efficiently design both positions and phases of the meta-atoms for a random metasurface.

The matrix pencil method has been employed to synthesize the far-field pattern of a sparse and random antenna array [39,40]. The method starts by sampling the desired pattern from the original periodic antenna array to form a Hankel matrix, then singular value decomposition (SVD) of this matrix is employed to obtain a reduced number of antenna elements, finally, matrix pencil method is utilized to reconstruct new element positions and excitations. Here, we employ the pencil method for synthesizing the far-field of random metalenses. Different from an antenna array, the meta-atoms on a metalens are passive. The amplitudes of the reflection/transmission coefficients are usually in a small range of variation e.g. [0.9, 1]. If we directly employing the original matrix pencil method, a large range excitation amplitude will occur. It leads to large side lobes, and results in a significantly reduced focusing efficiency.

To overcome this problem, we adapt the method pencil method, where the metasurface is divided into sections and each section is synthesized separately. Both 1D and 2D metalenses at 220 GHz with 85.2% and 78.2% focusing efficiencies are demonstrated with the proposed method. Minimum reduction of efficiency can be achieved with respect to the standard metalenses with periodic grids, while random metasurface will be polarization independent as demonstrated in [37,38]. Better performance of a metalens can be achieved comparing with the ones produced by the previously reported algorithm [37,38] with the same number of meta-atoms.

2. Matrix pencil method for random metasurface synthesis

We adopt the matrix pencil method previously for synthesis of non-uniform antenna arrays [39,40] for designing the random matasurface. We treat a 1D metasurface (along \hat{x} direction) as a linear antenna array, of which the far-field radiation pattern $F(\theta)$ can be expressed by

$$F(\theta) = \sum_{i=1}^{M} R_i e^{jkd_i \cos\theta}$$
(1)

where R_i is the complex excitation coeffcient of the *i*th meta-atom located at $x = d_i$, M is the total number of meta-atoms, k is the wavenumber, and θ is the angle between the far field vector and \hat{x} . The goal of the matrix pencil method here is to synthesize a new 1D metasurface with minimum number of randomly distributed meta-atoms while maintaining the same desired far-field pattern with a small tolerance. By letting $u = \cos\theta$ and $\omega_i = kd_i$, Eq. (1) can be written as

$$F(\cos^{-1}u) = \sum_{i=1}^{M} R_i e^{j\omega_i u}$$
(2)

To obtain the number of meta-atoms, their positions and excitation phases, the major steps of the proposed method are:

First, we sample the far-field pattern of the metasurface in Eq. (2) of u in [-1, 1] in uniform steps Δ satisfying Nyquist sampling criterion

$$\Delta \le \frac{\lambda}{d_{\max}} \tag{3}$$

where λ and d_{max} are wavelength and total length of the metasurface, respectively. In this paper, we choose $\Delta = 1/M$ [39]. At each sampling point, we have

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$$f(n) = F(\cos^{-1}n\Delta) = \sum_{i=1}^{M} R_i z_i^n$$
 (4)

where $z_i = e^{j\omega_i \Delta}$.

Second, we should construct a Hankel matrix with the sampling points in the radiation pattern

$$[Y] = \begin{bmatrix} y(0) & y(1) & \cdots & y(M) \\ y(1) & y(2) & \cdots & y(M+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(M) & y(M+1) & \cdots & y(2M) \end{bmatrix}$$
(5)

where y(n) = f(n-M). The number of the meta-atoms can be determined by SVD of the Hankel matrix

$$[Y] = [U][\Sigma][V]^{H}$$
(6)

The diagonal entries σ_i of the diagonal matrix $[\Sigma]$ are the singular values of [Y]. The columns of [U] and [V] matrices are the left- and right- singular vectors, respectively. In general, [Y] is a low-rank matrix, whose rank is smaller than its dimension. If we discard the singular values which is smaller than a certain threshold ε , we can rewrite Eq. (7) as

$$[Y_{\varrho}] = [U_{\varrho}][\Sigma_{\varrho}][V_{\varrho}]^{H}$$
⁽⁷⁾

The minimum number of meta-atoms Q can be obtained after discarding the small singular values

$$Q = \min\left\{q \left| \frac{\sqrt{\sum_{i=q+1}^{M} \sigma_i^2}}{\sqrt{\sum_{i=1}^{q} \sigma_i^2}} < \varepsilon \right\}$$
(8)

we choose $\varepsilon = 1 \times 10^{-3}$ in this work.

Third, the new positions of meta-atoms can be obtained by solving a generalized eigenvalue problem written below

$$([Y_{Q,f}] - z'[Y_{Q,l}])v = 0$$
(9)

where matrices $[Y_{Q,f}]$ and $[Y_{Q,f}]$ are obtained from $[Y_Q]$ by removing the first and last column, respectively. The parameters of z'_i are equal to the nonzero eigenvalues of z'. Once z'_i is obtained, the new positions of the meta-atoms can be computed by

$$d_i' = \frac{1}{jk\Delta} \ln(z_i') \tag{10}$$

where $j = \sqrt{-1}$. As noted in [39,41,42], we only use the real parts of d'_i . The estimated excitations can be obtained by substituting the new positions d'_i into Eq. (4)

$$R'_{i} = \left([\hat{Z}]^{H} [\hat{Z}] \right)^{-1} [\hat{Z}] F_{M}$$
(11)

where $\hat{z}'_i = \hat{z}' / |\hat{z}'|$

$$F_M = (f(-M), \cdots, f(M))^T$$
(12)

$$[\hat{Z}] = \begin{bmatrix} \hat{z}_{1}^{\prime-M} & \hat{z}_{2}^{\prime-M} & \cdots & \hat{z}_{Q}^{\prime-M} \\ \hat{z}_{1}^{\prime-M+1} & \hat{z}_{2}^{\prime-M+1} & \cdots & \hat{z}_{Q}^{\prime-M+1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{z}_{1}^{\prime M} & \hat{z}_{1}^{\prime M} & \cdots & \hat{z}_{Q}^{\prime M} \end{bmatrix}$$
(13)

We can find the least squares solution of excitations using Eq. (11), which leads to reduction of the error from discarding the imaginary part of the positions.

In contrast to a standard antenna array, where the excitation amplitude and phase can both be varied, the passive metalenses can only have the phase changed. When we employ the matrix pencil method to get a sparse and random metasurface, the computed excitation amplitude will be non-uniform, which is undesired for a lens.

A series of transmission metalenses working at 220 GHz with randomly distributed metaatoms are designed to demonstrate the proposed algorithm. As shown in Fig. 1, the metaatoms are high-index dielectric pillars (Si: $\varepsilon_r = 11.9$) embedded in a low-index dielectric slab (RT/duroid 5880: $\varepsilon_r = 2.2$, tan $\delta = 9 \times 10^{-4}$). The anti-reflective layers on top and bottom of the slab are made of RT/duroid 5880LZ ($\varepsilon_r = 1.96$, tan $\delta = 1.9 \times 10^{-3}$).

The simulated phase shifts and transmittance versus the radius of the Si pillar with $0.34\lambda_0$ period is plotted in Fig. 1, λ_0 is the wavelength in free space. It can be found that the metaatoms support 2π range of phase shift and simultaneously satisfy > 0.9 transmittance. The numerical aperture (NA) of the designed 1D and 2D metalenses is about 0.45.

As shown in Fig. 2, we separate the metalens into three sections which have Q_0 , Q_1 , and Q_2 number of meta-atoms, respectively. For each section, we can get the normalized array factor and the random meta-atom array can be synthesized by computing the excitation R'_i and position d'_i according to Eq. (1)-(11). The far field of the periodic metasurface array can be converted into a random array with a predefined tolerance. Figure 3 plots the far-field normalized array factor (AF) curves of the original periodic array with 149 meta-atoms, the random arrays with 111 and 86 meta-atoms when the matrix pencil method was directly employed, and random array with 111 meta-atoms when the matrix pencil method with sections was employed, respectively. The normalized AF in dB is defined as [43]

Normalized AF =
$$20 \log \left\{ \frac{|F(\theta)|}{|F_{\max}(\theta)|} \right\}$$
 (14)

where $F_{\max}(\theta)$ is the maximum value of array factor at $\theta = 0$. Those curves match with each other quite well, which indicates that the random arrays can well reconstruct the far field of a periodic one. As noted above, not only the reconstructed far field, but also the near field can affect the focusing efficiency of the random metalenses.



Fig. 1. Accumulated phase differences and transmissions versus the radius of Si pillar at 220 GHz, inset is configuration of the unit cell, which is designed with low index dielectric slab (RT/duroid 5880: $\varepsilon_r = 2.2$, $\tan \delta = 9 \times 10^{-4}$) patterned with high index dielectric pillars (Si: $\varepsilon_r = 11.9$), the antireflective layer on the top and bottom are RT/duroid 5880LZ($\varepsilon_n = 1.96$, $\tan \delta = 1.9 \times 10^{-3}$).

The excitation amplitudes of the passive meta-atoms are roughly the same (within the range of [0.9, 1], see in Fig. 1). We show the excitation amplitudes of the random arrays generated by the matrix pencil method with and without sections in Fig. 4. As shown in Fig. 4, the normalized excitation amplitudes are in the range of [0.5, 1.0] for the proposed method with 111 meta-atoms, while the range is [0, 1] for the same number when employed the matrix pencil method directly. In the design of 111 meta-atoms with the original matrix pencil method, the meta-atoms with amplitudes equal to 0 is outside of the antenna aperture and can be removed. Therefore, we can keep 86 meta-atoms, which will have the normalized excitation amplitudes be in the range of [0.4, 1.0]. In conclusion, with the proposed method, we can obtain a random array with amplitudes in a relative smaller range.



Fig. 2. Separate the full metasurface array into sections, each section is employed to sparse and reconstruct the far field, respectively. Q_0 , Q_1 , and Q_2 are the normalized array factor of each

sections in periodic array, Q'_0 , Q'_1 , and Q'_2 are the corresponding number of synthesized random metasurface meta-atoms with matrix pencil method, respectively.

3. Results and discussions

In this section, we will discuss how to choose a proper series of sparse factors Q_i to design a high efficiency metalens. For the 1D metalens with 149 meta-atoms in Sec. 2. Taking advantage of the central symmetry of the metalens, only half of the meta-atoms are required

to be synthesized. We separate the original periodic array into $Q_0 = 20$, $Q_1 = 25$, and $Q_2 = 30$ from the side to center of metalens as shown in Fig. 2. According to our experience, reducing the number of meta-atoms in the side will have a large impact on focusing efficiency. In this work, the matrix pencil method was only used on the parts where $Q_1 = 25$ and $Q_2 = 30$. The minimum number of random meta-atoms is determined by Eq. (8), while the maximum number is determined by the antenna array aperture size. The numbers of randomly distributed meta-atoms for the two sections Q_1' and Q_2' are ranging from 14 to 22 and from 14 to 25, respectively. We list the focusing efficiencies with respect to the number of random meta-atoms as shown in Table 1 and Table 2. The purpose of this is to obtain the highest focusing efficiency while with minimum number of meta-atoms. The error in Table 1 to Table 3 is defined as the aberration between the designed and simulated focal lengths.

We chose $Q_0 = 20$, $Q_1 = 17$, and $Q_2 = 19$ to generate the random metalens (Fig. 5 (a)). After reconstructing the far field, we simulated the intensity distributions of the periodic and random metalenses (Fig. 5 (b) and (c)). We obtained the focusing efficiencies to be 86.7% and 85.7%, respectively. And the full width at half maximum (FWHM) of normalized intensity at the focal plane is 1.50 ± 0.01 mm (Fig. 5 (d) and (e)), with respect to the diffraction limit of 1.51 mm. We also show the normalized intensity at the focal plane for random metalens with 86 meta-atoms when synthesized uniformly in Fig. 5 (f). As predicted, strong side lobes are observed, which reduces dramatically the focusing efficiency. In order to show the advantage of the proposed matrix pencil method for synthesizing random metasurfaces, we list the focusing efficiencies of 10 kinds of arrays (denoted as 1 to 10 in Table 3) by distributing 111 meta-atoms using the previously reported algorithm [37,38] in a same aperture. We can see that the average efficiency is 69.6%, which is much smaller than the metalens designed by the proposed algorithm.



Fig. 3. Normalized array factor (AF) of the periodic and synthesized random metasurface antenna array with matrix pencil method directly and separately. The original periodic array is with 149 meta-atoms, the number of random meta-atoms are 111 and 89 when synthesized directly, and are 111 when synthesized separately.



Fig. 4. Normalized excitation amplitude of the metasurface array antenna when obtain the random matrix with uniformly and separately sparse strategies, respectively.

Table 1. Focusing efficiencies of the second part (25 number of meta-atoms) of a series of random metalens different number of random meta-atoms (Q_1) from the matrix pencil method

Q_1'	22	21	20	19	18	17	16	15	14
Error (λ_0)	0.9	0.9	0.9	1.0	0.9	1.0	0.9	1.0	0.9
Focusing efficiency	86.8%	86.2%	85.9%	86.1%	86.8%	86.6%	82.8%	84.6%	82.9%

Table 2. Focusing efficiencies of the second part (30 number of meta-atoms) of a series of random metalens different number of random meta-atoms (Q_2) from the matrix pencil method

Q_2'	25	24	23	22	21	20	19	18	17
Error (λ_0)	0.1	0.5	0.1	0.2	0.04	0.04	0.04	0.04	0.3
Focusing efficiency	81.6%	85.1%	86.8%	86.1%	79.5%	85.2%	86.7%	82.1%	85.5%

Table 3. Focusing efficiencies of 10 kinds of random metalens with 111 number of metaatoms obtained by full random algorithm, respectively

Random array	1	2	3	4	5	6	7	8	9	10
Error (λ_0)	0.5	0.6	1.1	1.1	0.5	0.9	1.1	0.6	1.0	0.6
Focusing efficiency	73.6%	71.6%	62.7%	70.2%	73.6%	70.1%	64.4%	69.9%	71.1%	69.1%

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Fig. 5. Simulated power intensity of the periodic and random metasurface antenna array (a), the focusing efficiencies are 86.7% and 85.7% for periodic (b) and random (c) array, normalized intensity of the focusing field for (d) periodic and (e) random array with 111 metaatoms when synthesized separately (f) random array with 86 meta-atoms when synthesized uniformly.





We also demonstrated a 2D random metalens converted from a periodic metalens with a 100×100 array of meta-atoms (Fig. 6). Only quarter of the full array was synthesized with the proposed method. The other parts were obtained due to the symmetry. The focusing efficiencies for the random and periodic metalenses are 78.2% and 81.5%, respectively. The FWHM of the intensity distributions of the random metalens at the focal plane with x and y polarizations, respectively, are shown in Fig. 6 (c). It clearly shows that our designed random metalens is polarization independent.

4. Conclusion

We presented the matrix pencil method for synthesis of random metalenses. In order to design high-efficiency random metalenses, the idea is to reconstruct well the far field with excitation amplitude of the meta-atoms, to archive better near field focusing reconstruction. The metaatom array is first divided into several sections, and the matrix pencil method is employed in sections separately. The relationship between the focusing efficiency and the number of random meta-atoms as well as the excitation amplitude are discussed in detail. A random

metalens with focusing efficiency of 85.7% is designed. The simulation shows that it is much higher than the random meta-atoms designed by the previously reported algorithm [37,38].

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References

- 1. N. Yu and F. Capasso, "Flat optics with designer metasurfaces," Nat. Mater. 13(2), 139-150 (2014).
- S. B. Glybovski, S. A. Tretyakov, P. A. Belov, Y. S. Kivshar, and C. R. Simovski, "Metasurfaces: from microwaves to visible," Phys. Rep. 634, 1–72 (2016).
- S. Chang, X. Guo, and X. Ni, "Optical metasurfaces: progress and applications," Annu. Rev. Mater. Res. 48(1), 279–302 (2018).
- C. L. Holloway, E. F. Kuester, J. A. Gordon, J. O'Hara, J. Booth, and D. R. Smith, "An overview of the theory and applications of metasurfaces: the two-dimensional equivalents of metamaterials," IEEE Antennas Propag. Mag. 54(2), 10–35 (2012).
- A. Monti, J. Soric, A. Alu, F. Bilotti, A. Toscano, and L. Vegni, "Overcoming mutual blockage between neighboring dipole antennas using a low-profile patterned metasurface," IEEE Antennas Wirel. Propag. Lett. 11(5), 1414–1417 (2015).
- H. Li, G. Wang, H. X. Xu, T. Cai, and J. Liang, "X-band phase-gradient metasurface for high-gain lens antenna application," IEEE Trans. Antenn. Propag. 63(11), 5144–5149 (2015).
- F. H. Lin and Z. N. Chen, "Low-profile wideband metasurface antennas using characteristic mode analysis," IEEE Trans. Antenn. Propag. 65(4), 1706–1713 (2017).
- W. E. I. Liu, Z. N. Chen, and X. Qing, "Compact wideband metasurface-based circularly polarized antenna for Ka-band phased array," (IEEE, 2017) in Proceedings of Antennas and Propagation in Wireless Communications, 17-20.
- F. H. Lin and Z. N. Chen, "A method of suppressing higher order modes for improving radiation performance of metasurface multiport antennas using characteristic mode analysis," IEEE Trans. Antenn. Propag. 66(4), 1894– 1902 (2018).
- Y. Liu, K. Li, Y. Jia, Y. Hao, S. Gong, and Y. J. Guo, "Wideband RCS reduction of a slot array antenna using polarization conversion metasurfaces," IEEE Trans. Antenn. Propag. 64(1), 326–331 (2016).
- Y. Zhao, J. Gao, X. Cao, T. Liu, L. Xu, X. Liu, and L. Cong, "In-band RCS reduction of waveguide slot array using metasurface bars," IEEE Trans. Antenn. Propag. 65(2), 943–947 (2017).
- Y. Zhao, X. Cao, J. Gao, X. Yao, T. Liu, W. Li, and S. Li, "Broadband low-RCS metasurface and its application on antenna," IEEE Trans. Antenn. Propag. 64(7), 2954–2962 (2016).
- D. F. Sievenpiper, "Nonlinear grounded metasurfaces for suppression of high-power pulsed RF currents," IEEE Antennas Wirel. Propag. Lett. 10, 1516–1519 (2011).
- A. Li, S. Kim, Y. Luo, Y. Li, J. Long, and D. F. Sievenpiper, "High-power transistor-based tunable and switchable metasurface absorber," IEEE Trans. Microw. Theory Tech. 65(8), 2810–2818 (2017).
- X. Liu, K. Fan, I. V. Shadrivov, and W. J. Padilla, "Experimental realization of a terahertz all-dielectric metasurface absorber," Opt. Express 25(1), 191–201 (2017).
- P. Y. Chen and A. Alu, "Mantle cloaking using thin patterned metasurfaces," Phys. Rev. B 84(20), 205110 (2011).
- X. Ni, Z. J. Wong, M. Mrejen, Y. Wang, and X. Zhang, "An ultrathin invisibility skin cloak for visible light," Science 349(6254), 1310–1314 (2015).
- S. Liu, H. X. Xu, H. C. Zhang, and T. J. Cui, "Tunable ultrathin mantle cloak via varactor-diode-loaded metasurface," Opt. Express 22(11), 13403–13417 (2014).
- Z. H. Jiang and D. H. Werner, "Dispersion engineering of metasurfaces for dual-frequency quasi-threedimensional cloaking of microwave radiators," Opt. Express 24(9), 9629–9644 (2016).
- R. Wang, B. Z. Wang, Z. S. Gong, and X. Ding, "Creation of an arbitrary electromagnetic illusion using a planar ultrathin metasurface," IEEE Photonics J. 9(4), 1–9 (2017).
- X. Ni, A. V. Kildishev, and V. M. Shalaev, "Metasurface holograms for visible light," Nat. Commun. 4(1), 2807 (2013).
- G. Zheng, H. Mühlenbernd, M. Kenney, G. Li, T. Zentgraf, and S. Zhang, "Metasurface holograms reaching 80% efficiency," Nat. Nanotechnol. 10(4), 308–312 (2015).
- X. Ni, S. Ishii, A. V. Kildishev, and V. M. Shalaev, "Ultra-thin, planar, Babinet-inverted plasmonic metalenses," Light Sci. Appl. 2(4), e72 (2013).
- M. Khorasaninejad, W. T. Chen, R. C. Devlin, J. Oh, A. Y. Zhu, and F. Capasso, "Metalenses at visible wavelengths: Diffraction-limited focusing and subwavelength resolution imaging," Science 352(6290), 1190– 1194 (2016).

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- W. T. Chen, A. Y. Zhu, V. Sanjeev, M. Khorasaninejad, Z. Shi, E. Lee, and F. Capasso, "A broadband achromatic metalens for focusing and imaging in the visible," Nat. Nanotechnol. 13(3), 220–226 (2018).
- 26. S. Wang, P. C. Wu, V.-C. Su, Y.-C. Lai, M.-K. Chen, H. Y. Kuo, B. H. Chen, Y. H. Chen, T.-T. Huang, J.-H. Wang, R.-M. Lin, C.-H. Kuan, T. Li, Z. Wang, S. Zhu, and D. P. Tsai, "A broadband achromatic metalens in the visible," Nat. Nanotechnol. 13(3), 227–232 (2018).
- 27. Y. Li, X. Li, M. Pu, Z. Zhao, X. Ma, Y. Wang, and X. Luo, "Achromatic flat optical components via compensation between structure and material dispersions," Sci. Rep. 6(1), 19885 (2016).
- N. Yu, P. Genevet, M. A. Kats, F. Aieta, J. P. Tetienne, F. Capasso, and Z. Gaburro, "Light propagation with phase discontinuities: generalized laws of reflection and refraction," Science 334(6054), 333–337 (2011).
- X. Ni, N. K. Emani, A. V. Kildishev, A. Boltasseva, and V. M. Shalaev, "Broadband light bending with plasmonic nanoantennas," Science 335(6067), 427 (2012).
- Y. F. Yu, A. Y. Zhu, R. Paníagua-Domínguez, Y. H. Fu, B. Luk'yanchuk, and A. I. Kuznetsov, "Hightransmission dielectric metasurface with 2^π phase control at visible wavelengths," Laser Photonics Rev. 9(4), 412–418 (2015).
- A. Arbabi, Y. Horie, M. Bagheri, and A. Faraon, "Dielectric metasurfaces for complete control of phase and polarization with subwavelength spatial resolution and high transmission," Nat. Nanotechnol. 10(11), 937–943 (2015).
- 32. H. Yi, S. W. Qu, K. B. Ng, C. H. Chan, and X. Bai, "3–D printed millimeter-wave and terahertz lenses with fixed and frequency scanned beam," IEEE Trans. Antenn. Propag. 64(2), 442–449 (2016).
- D. Isakov, C. J. Stevens, F. Castles, and P. S. Grant, "3D-printed high dielectric contrast gradient index flat lens for a directive antenna with reduced dimensions," Advanced Materials Technologies 1(6), 1600072 (2016).
- S. Jahani and Z. Jacob, "All-dielectric metamaterials," Nat. Nanotechnol. 11(1), 23–36 (2016).
 S. J. Byrnes, A. Lenef, F. Aieta, and F. Capasso, "Designing large, high-efficiency, high-numerical-aperture,"
- transmissive meta-lenses for visible light," Opt. Express **24**(5), 5110–5124 (2016).
- L. Hsu, M. Dupré, A. Ndao, J. Yellowhair, and B. Kanté, "Local phase method for designing and optimizing metasurface devices," Opt. Express 25(21), 24974–24982 (2017).
- M. Dupré, L. Hsu, and B. Kanté, "On the design of random metasurface based devices," Sci. Rep. 8(1), 7162 (2018).
- M.Dupré, J. Park, and B. Kanté, "A random metasurface for an all polarization flat lens," in Proceedings of CLEO: Science and Innovations (OSA, 2018), JTu5A–38.
- 39. Y. Liu, Z. Nie, and Q. H. Liu, "Reducing the number of elements in a linear antenna array by the matrix pencil method," IEEE Trans. Antenn. Propag. **56**(9), 2955–2962 (2008).
- Y. Liu, Q. H. Liu, and Z. Nie, "Reducing the number of elements in the synthesis of shaped-beam patterns by the forward-backward matrix pencil method," IEEE Trans. Antenn. Propag. 58(2), 604–608 (2010).
- 41. T. K. Sarkar and O. Pereira, "Using the matrix pencil method to estimate the parameters by a sum of complex exponentials," IEEE Antennas Propag. Mag. **37**(1), 48–55 (1995).
- E. K. Miller and D. M. Goodman, "A pole-zero modeling approach to linear array synthesis I: the unconstrained solution," Radio Sci. 18(1), 57–69 (1983).
- 43. W. L. Stutzman and G. A. Thiele, Antenna Theory and Design, 3rd edition, (John Wiley & Sons, 2012).